



## Mathematical proof difficulties among mathematics education students: A cognitive perspective

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### Abstract

This study aims to analyze students' difficulties in mathematical proof, identify the causes from a cognitive perspective, and explore solutions that can overcome them. This study uses a qualitative descriptive approach to analyze students' difficulties in mathematical proof from a cognitive perspective. Data were collected from 22 new Mathematics Education students through tests, questionnaires, observations, and interviews. Data analysis used the Miles and Huberman interactive model, including data collection, data reduction, data presentation, and drawing conclusions. This study found that students' difficulties in mathematical proof can be classified into three main categories of executive functions: working memory, inhibitory control, and switching ability. Obstacles in working memory cause students to have difficulty storing and processing logical information simultaneously, indicated by errors in arranging steps, remembering logical rules, and connecting relevant concepts. Obstacles in inhibitory control are seen from impulsive actions, anxiety during exams, and the inability to resist the urge to conclude prematurely. Meanwhile, weak switching ability makes students inflexible in switching strategies, relying too much on truth tables, and having difficulty integrating information into logical arguments. These three executive functions are related to the activity of the prefrontal cortex, parietal lobe, hippocampus, basal ganglia, and amygdala. Recommended cognitive-based solutions include chunking strategies, multimodal approaches, reflection-based exercises, and emotional regulation through mindfulness to improve flexibility of thinking and effectiveness of mathematical proof. Students' mathematical proof difficulties are related to the limitations of executive functions, namely working memory, inhibitory control, and switching ability. Therefore, a cognitive-based approach is needed to improve logical understanding and mathematical thinking skills systematically and flexibly.

**Keywords:** Mathematical proof, executive functions, cognitive, mathematics education student, cognitive strategies

### Introduction

Mathematics is taught from elementary school to college to equip individuals with logical and analytical thinking skills (Marni & Pasaribu, 2021). Mathematics is a complex field that requires systematic thinking <sup>[1]</sup>. In line with research <sup>[2]</sup>, mathematics is taught to teach logical, analytical, systematic, critical, and creative thinking skills as well as the ability to work together. Therefore, mathematics has an important role in education because it trains critical thinking, reasoning, problem solving, and understanding concepts in depth.

National Council of Teachers of Mathematics <sup>[3]</sup> put forward five basic mathematical skills, including problem solving, reasoning and proof, communication skills, connection skills and representation skills <sup>[4, 5, 6]</sup>. One of the basic skills that students must master is the ability to reason and prove (reasoning and proof). Mathematical reasoning ability is a logical and systematic thinking skill that allows someone to draw new conclusions based on existing facts or statements <sup>[7]</sup>. While mathematical proof is an integral part of mathematical reasoning <sup>[8]</sup>. This is the process used to show the truth of a mathematical statement, either to support or refute it <sup>[9]</sup>.

According to that mathematical reasoning and proof skills are fundamental aspects in mathematics learning that train critical thinking, generalization, and deep understanding of concepts. <sup>[10]</sup> emphasizes that these skills are important for

logical thinking, effective communication, and are relevant in technology-based careers and everyday life. Therefore, mathematics learning must emphasize strengthening reasoning and proof so that students are able to solve problems in a structured manner.

Although mathematical proof is important, the results of observations and interviews by researchers show that many students still experience difficulties, such as determining the initial steps, understanding logical symbols and sets, and designing systematic proofs. This finding is supported by research <sup>[12]</sup> and <sup>[13]</sup>, which revealed students' weaknesses in connecting facts, using definitions as the basis for arguments, and mastering mathematical proofs. In other words, although these abilities are very important for solving mathematical problems, students still have not achieved an adequate level of mastery. Furthermore, in observations during the Mathematics Learning Practice (PPM) in the Logic and Set Theory course for one semester, it was found that most Mathematics Education students at Sanata Dharma University in the 2024/2025 academic year experienced obstacles in compiling and understanding mathematical proofs in a coherent and systematic manner.

Students' difficulties in mathematical proof can be categorized into several types of errors. Lerner <sup>[13]</sup> called mathematics learning difficulties dyscalculia, which is related to central nervous system disorders. Kastolan <sup>[14]</sup> divides errors into conceptual errors, where students

misinterpret or use terms and principles, and procedural errors, which are inaccuracies in compiling steps to solve problems. Lerner <sup>[15]</sup> also identified several characteristics of mathematics learning difficulties, such as disturbances in spatial relationships, visual perception abnormalities, and difficulty recognizing and understanding symbols. Lestari <sup>[16]</sup> emphasized that mathematical proof skills include understanding symbols and compiling evidence based on definitions and theorems. <sup>[17]</sup> emphasizes the importance of understanding students' difficulties in constructing mathematical proofs. Knuth <sup>[16]</sup> also 3 emphasized that proof is a central aspect of mathematics learning. Therefore, this ability should be a focus in mathematics education.

Based on the analysis of the results of the 2023/2024 academic year final exams for the odd semester, it was found that students still experience difficulties in mathematical proof in the Logic and Set Theory course. Some of the obstacles that arise include difficulties in designing strong proof arguments, the tendency to use algebraic manipulations without understanding the basics of logic, and a lack of understanding of the basic concepts of logic and sets. Students' answers show that many of them have not been able to construct proofs with the correct flow, either directly or indirectly. In addition, understanding of the notation and logical rules needed to construct valid proofs is still weak. This can happen due to a lack of strong understanding of the basic concepts of logic and sets and skills in applying these concepts concretely. This difficulty shows that there are gaps in learning that need to be addressed, both in terms of teaching methods and learning approaches used.

Students often have difficulty in solving proof problems even though they have mastered the necessary mathematical computational skills. This is due to the need to identify relevant numerical and linguistic information in the problem, and integrate it into a logical argument structure <sup>[18, 19, 20]</sup>. This thinking process occurs internally in the human brain <sup>[21]</sup>, and to understand the steps of students' thinking in mathematical proof, an approach is needed that can stimulate their thinking process more effectively <sup>[22]</sup>. In this context, cognitive provides a framework for understanding how the mind works in constructing mathematical proofs. This science includes the fields of cognitive psychology, linguistics, and cognitive neuroscience, which together explain how memory, attention, inhibitory control, and flexibility of thinking work in solving complex tasks such as mathematical proofs. <sup>[23]</sup> adding that critical thinking, which involves good reasoning skills, requires support from right-brain activity in the process, for example by involving emotional elements.

Research in cognitive perspective shows that mathematical thinking involves multiple areas of the brain. The prefrontal cortex is known to play a role in complex executive functions, including working memory and inhibitory control. This function is important in mathematical processing because it allows individuals to temporarily store and manipulate information, as well as inhibit irrelevant responses <sup>[24, 25, 26]</sup>. However, in the context of education, cognitive- based approaches are more focused on how information is mentally processed and how learning strategies can be tailored to support the work of these cognitive systems <sup>[27, 28]</sup>.

Research by <sup>[29]</sup> shows that high-level mathematical reasoning involves brain circuits related to numbers and space, not just language processing. The areas involved include the intraparietal parietal lobe, bilateral prefrontal cortex, and inferior temporal region, which are associated with number intuition and mental manipulation of spatial objects. <sup>[30]</sup> states that the prefrontal cortex is active in solving problems, which functions as an executive control center for high-level thinking. Suyadi added that the prefrontal cortex also plays a role in solving problems, controlling emotions, and determining personality <sup>[27]</sup>. However, many educators do not yet understand how cognitive processes in the brain affect the learning process, even though this understanding can be used to optimize learning strategies to improve students' understanding of complex mathematical concepts. By understanding the principles of executive function from a cognitive perspective, educators can design more targeted interventions to help students improve their logical thinking skills, especially in the context of mathematical proof.

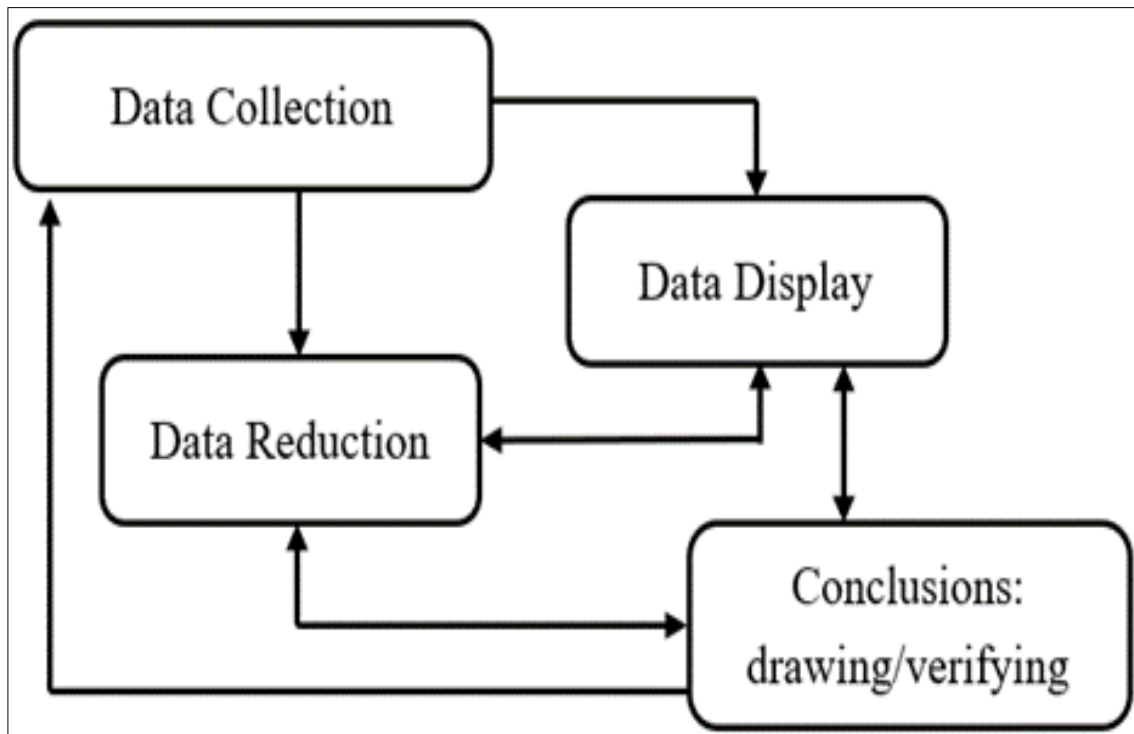
Based on various findin students still experience significant difficulties in constructing mathematical proofs logically and systematically. On the other hand, cognitive-based approaches that explain how executive functions such as working memory, inhibitory control, and switching ability work have not been widely utilized in learning. Therefore, this study aims to analyze students' difficulties in mathematical proofs, identify their causes from a cognitive perspective, and explore strategic solutions to overcome them through a cognitive approach.

## Methodology

1. **Research design:** This study uses a qualitative research design with a qualitative descriptive approach. This study focuses on analyzing the difficulties faced by Mathematics Education students in mathematical proof, viewed from a cognitive perspective.
2. **Participants:** The selection of interview subjects was carried out by purposive sampling, selecting students who showed consistent error patterns for in-depth and useful analysis <sup>[31]</sup>.
3. **Instruments**  
The instruments used in this study include
  - Observation Sheet to find out in general the difficulties of mathematical proof of the object being studied.
  - The test questions consist of 2 Mid-Semester Exam (UTS) questions on Logic and Set Theory for the odd semester of 2024/2025 which are adjusted to the research needs of researchers.
  - Validation sheet of the questionnaire sheet from the aspects of sentence clarity, content accuracy, relevance, content validity, and language accuracy.
  - Questionnaire on students' mathematical proof difficulties from a cognitive perspective.

## Data Analysis

The data analysis model used is Miles and Huberman's interactive analysis <sup>[32]</sup>. The data analysis technique flow is shown in Figure.



**Fig 1** Miles and Huberman Data Analysis Process

1. **Data Collection:** Collecting primary data from mid-term exam results as well as supporting data from questionnaires and interviews to identify student difficulties in mathematical proof.
2. **Data Reduction:** Selecting and summarizing data relevant to the research focus, grouping errors based on certain categories.
3. **Data Display:** Displaying the reduced data in descriptive form for further analysis to answer the problem formulation.
4. **Drawing Conclusions:** Formulating findings based on data analysis by connecting difficulties, causes, and solutions from a cognitive perspective.

### Analysis and Interpretation of data

#### Results

##### Data collection

This study collected data through two main methods: observation and the Logic and Set Theory Mid-Semester Exam (UTS) in the odd semester of 2024/2025. Observations were carried out in the classroom to generally observe students' difficulties in conducting mathematical proofs. The main data of this study came from the results of the Logic and Set Theory Mid-Semester Exam (UTS) in the odd semester of 2024/2025, which were collected after students completed the exam. In addition, supporting data for this study were obtained through questionnaires and interviews. The questionnaire was distributed online to students using Google Form to identify the thinking errors and proof strategies they used. In-depth interviews were conducted with a number of selected subjects to confirm the findings from the questionnaire and observation results, as well as to explore more deeply the cognitive factors that

influence students' thinking processes in constructing formal logical proofs. In this study, students' difficulties in proofs were classified based on the cognitive perspective, specifically referring to the executive function theory proposed by Haan<sup>[33]</sup>. Executive function is divided into three main components, namely working memory, inhibitory control, and switching ability or cognitive flexibility. Working memory refers to the ability to store and process information temporarily in the mind; inhibitory control is the ability to restrain impulsive responses and ignore irrelevant distractions; and switching ability is the ability to move flexibly between tasks, rules, or thinking strategies<sup>[33]</sup>.

##### Data Reduction

Based on the results of the Mid-Semester Exam, students showed various difficulties in understanding and compiling mathematical proofs that can be classified into three main categories based on executive functions, namely working memory, inhibitory control, and switching ability.

The first difficulty is related to working memory. Many students experience high working memory load, especially when they have to store and manipulate logical information simultaneously. This is indicated by errors in simplifying logical expressions, repeating statements without additional information, or forgetting previously designed proof steps. These errors indicate the limited capacity of working memory in maintaining complex and logically connected information during the mathematical thinking process. The second difficulty is related to inhibitory control, which is the inability to inhibit automatic or impulsive responses that are not in accordance with the context of the proof. Some students rush to complete the proof without conducting in-depth analysis or adding irrelevant steps. In addition, cognitive pressure such as anxiety also interferes with their ability to think systematically, as seen from cases where

students leave blank answers or stop in the middle of the proof because they feel unsure or afraid of being wrong. The third difficulty is related to switching ability, which is cognitive flexibility in switching between strategies or logical representations. Students who experience obstacles in this aspect tend to be stuck in one procedural approach,

such as using truth tables, even though the context of the problem requires abstract and symbolic thinking. They also show difficulties in integrating logical information into coherent arguments, as well as in relating old knowledge to new contexts, such as when interpreting subset relations or applying basic rules of logic in formal symbolic form.

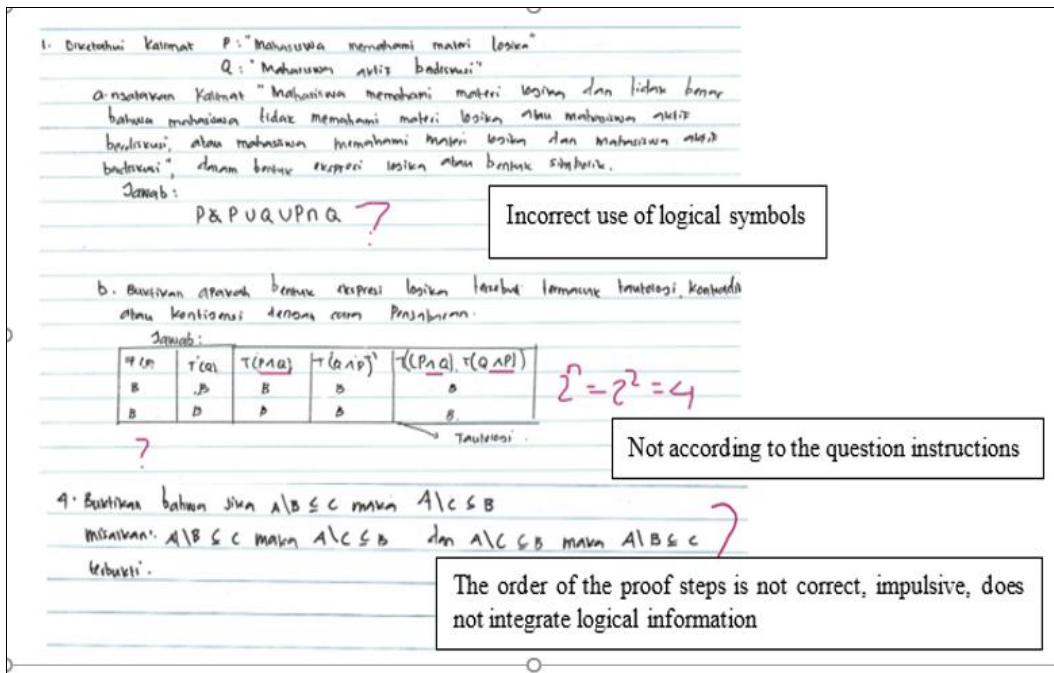


Fig 2: Students' Mathematical Proof Answers

The questionnaire results also confirmed the difficulties experienced by students based on three categories of executive functions. The following table shows the

percentage distribution of students' answers related to these difficulties.

Table 1: Results of the Mathematical Proof Difficulty Questionnaire

Answer categories	Percentage
Often	29.55%
Sometimes	69.89%
Never	0.56%

The questionnaire results confirmed that 99.44% of students experienced difficulties in mathematical proof.

Interview results confirmed that students faced various difficulties in mathematical proofs, such as understanding logical symbols, remembering steps, determining the order of proofs, and identifying errors. Some students stated, "Misunderstanding the symbols," or "It is difficult to remember the steps of the proof." Other difficulties included integrating the steps of the proof and connecting previous concepts to new concepts, as expressed, "In the next step, I continue to connect the elements," or "What was learned in class and what was done on the problem is difficult to connect." The majority of students also preferred truth tables over the explanation method, as stated, "Truth tables are easier," or "If I had to choose, I would use truth tables." In addition, exam anxiety also became a barrier, "Avoiding solving problems if there are other problems, try another one first."

Data Display

1. Difficulties Related to Working Memory

The results of the Mid-Semester Exam showed that many students had difficulty in storing and processing logical information simultaneously during the proof. This error was seen from the inability to simplify logical expressions, forgetting to include logical rules, repeating statements without additional new information, and failing to connect the steps of the proof coherently. This difficulty reflects the limitations of working memory capacity caused by impaired dorsolateral prefrontal cortex (DLPFC) function and weak integration with the parietal lobe, which plays a role in logical and spatial processing [34, 35, 36]. Furthermore, students also had difficulty in linking old concepts with new information, such as when applying subset relations and difference set operations in the context of proof. This difficulty indicates weak memory consolidation and semantic integration, which are influenced by dysfunction

of the hippocampus and inferior parietal lobe [37,38]. Research [39, 40, 41] shows that limited working memory capacity has a significant impact on performance in complex mathematical tasks.

## 2. Difficulties Associated with Inhibitory Control

Another significant error is related to students' failure to control impulses or inhibit irrelevant automatic responses. Some students act impulsively in simplifying logic, adding unnecessary steps, or jumping to conclusions without following the logical process. In addition, cognitive pressure such as anxiety causes students to lose focus, feel afraid of being wrong, and even leave the problem unanswered. This difficulty is closely related to inhibition in the ventromedial prefrontal cortex (vmPFC) and the dominance of amygdala activity, which interferes with decision-making and emotion regulation [42, 43, 44]. High amygdala activity is known to reduce working memory capacity [45, 46], and negatively impact the efficiency of the Default Mode Network (DMN) and Fronto-Parietal Network (FPN) functions in regulating cognition [47]. These findings support the importance of training emotion regulation and metacognitive strategies in the context of logic and mathematics learning [48].

## 3. Difficulties Related to Switching Ability

Students also showed difficulties in switching between strategies or approaches to proof. Many relied on one method, such as the use of truth tables, even though this method was not relevant to the type of problem. This indicates a barrier in thinking flexibility and strategy adaptation. Students also had difficulty in integrating logical information as a whole, resulting in incoherent or contradictory arguments. This difficulty is related to poor coordination between the dorsolateral prefrontal cortex, parietal lobe, and salience network. This network is responsible for detecting important elements, directing attention, and allocating cognitive resources for strategy switching [49, 50, 51]. On the other hand, the dominance of procedural processing in the intraparietal sulcus (IPS) and angular gyrus (AG) and recurrent activity in the basal ganglia also limit adaptation to new, more abstract approaches [52, 53, 54]. The lack of prefrontal cortex activation hinders information integration and rule-based step-making [55, 56].

## Discussion

### 1. Difficulties Related to Working Memory

Students' difficulties in maintaining and processing logical information simultaneously indicate limitations in working memory function. Errors such as forgetting proof steps, repeating statements without additional information, or failing to include the logical rules used reflect a working memory load that exceeds capacity. This is in line with the working memory model by Baddeley & Hitch (1974) that central executive limitations cause failure in managing complex information. The dorsolateral prefrontal cortex (DLPFC) responsible for working memory can experience overload when processing abstract elements [33], exacerbated by impaired interaction with the medial parietal cortex [58]. Cognitive Load Theory [59] explained that the high intrinsic cognitive load in mathematical proofs can cause students to

forget steps or repeat irrelevant arguments. In addition, disruption of the Default Mode Network (DMN) weakens self-reflection and argument evaluation [41]. Consequently, students are unable to examine the relationships between steps or correct repeated errors. To overcome this problem, strategies such as chunking [60], the use of flashcards can increase information retention [61], repetitive exercises with gradual questions can strengthen cognitive endurance and increase working memory efficiency [62], and visualization of concepts with concept maps [63, 64] has been shown to strengthen working memory and increase the fluency of logical thinking. Contextual Teaching and Learning Approach [65], it is also useful to relate new information to prior knowledge through concrete experiences.

### 2. Difficulties Associated with Inhibitory Control

Students' difficulties in controlling impulsive urges when constructing proofs indicate weak inhibitory control functions. Students often rush to conclusions, add irrelevant steps, or even stop proofs due to anxiety and uncertainty. These difficulties indicate low involvement of the ventromedial prefrontal cortex (vmPFC) and anterior cingulate cortex (ACC), which are structurally responsible for decision-making and error detection [42, 66]. In addition, excessive amygdala activity when facing cognitive pressure also suppresses working memory capacity and disrupts emotional regulation [45, 46]. Neural Network Theory [67] also highlighted DMN disorders that reduce self-reflection and decision-making efficiency [47]. To improve inhibitory control function, mindfulness-based approaches such as meditation and breathing exercises are effective in reducing anxiety [68, 69, 70] while a positive approach to mistakes and assessment simulations help students be better prepared for academic pressure [71,72].

### 3. Difficulties Related to Switching Ability

Students' difficulties in switching from one strategy or form of representation to another indicate low cognitive flexibility. Many students rely on one approach such as truth tables, even though the method is not always relevant to the context of the problem. This difficulty is also seen from the inability to integrate logical information into a coherent argument and the failure to connect old concepts to new contexts. This switching function obstacle is related to weak coordination between the dorsolateral prefrontal cortex, parietal lobe, and salience network which play a role in detecting and shifting attention to important elements in the proof process [49, 50, 51]. In addition, the dominance of activity in the basal ganglia and intraparietal sulcus (IPS) indicates a procedural tendency that inhibits cognitive flexibility [52, 54]. To improve switching ability, learning strategies such as cognitive apprenticeship [73], multimodal approach [74], counter-intuitive experiments [75] can help students adjust their thinking strategies to the demands of the problem, and proofreading strategies [76] are also effective in improving cognitive flexibility and abstract understanding. To address this, educators can use visual aids, step-by-step simulations, and case studies to facilitate the organization of information [77, 78, 79, 80]. Analogy-based and narrative approaches can also strengthen critical thinking skills and the relationships between logical elements [81, 82]. While the use of concept

maps and proof journals helps develop executive functions and ensures that proof steps are met<sup>[83, 84]</sup> also effective in sharpening the flexibility of logical thinking.

### Conclusion

This study shows that the difficulties of Mathematics Education students in mathematical proof are related to obstacles in three main executive functions: working memory, inhibitory control, and switching ability. These three executive functions show a close relationship with various brain areas such as the prefrontal cortex, parietal lobe, hippocampus, basal ganglia, and amygdala and influence each other in the process of mathematical proof. Suboptimal information integration, strategy selection, decision making, and emotional regulation hinder students' logical thinking processes.

Students are advised to train their thinking flexibility and abstract understanding through educational technology and step-by-step exercises, while educators can integrate neuroscience approaches into learning design. Parental support is also needed in creating a learning environment that supports the development of executive functions, including support for emotional regulation and logical exercises. Further research is suggested to involve cross-disciplinary collaboration and the use of technologies such as fMRI or EEG to empirically explore neural activity and more accurate results. in the context of mathematical proof.

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